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On the solution of Large Scale Bi-Level Linear Vector Optimization Problems through TOPSIS

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Abstract- *In this paper, we extend TOPSIS (Technique for Order Preference by Similarity Ideal Solution) for solving Large Scale Bi-level Linear Vector Optimization Problems (LS-BL-LVOP). In order to obtain a compromise (satisfactory) solution to the LS-BL-LVOP problems using the proposed TOPSIS approach, a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS) are proposed and modeled to include all the objective functions of both the first and the second levels. An interactive decision making algorithm for generating a compromise (satisfactory) solution through TOPSIS approach is provided where the first level decision maker (FLDM) is asked to specify the relative importance of the objectives. Finally, a numerical example is given to clarify the main results developed in the paper.*

Keywords- *Decision making; Vector Optimization Problems; Bi-level programming; TOPSIS; block angular structure; large scal programming; fuzzy programming.*

1. INTRODUCTION:

The increasing complexity of modern-day society has brought new problems involving very large numbers of variables and constraints. Due to the high dimensionality of the problems, it becomes difficult to obtain optimal solutions for such large scale programming (LSP) problems. Fortunately, however, most of the LSP problems arising in application almost always have a special structure that can be exploited. One familiar structure is the block angular structure to the constraints that can be used to formulate the subproblems [31, 47, 56].

After the publication of the Dantzig-Wolfe decomposition method [31], the subsequent works on large scale linear and nonlinear programming problems with block angular structure have been numerous (see f. i. [13, 15, 16, 18, 34,35, 40, 49,54]).

The decentralized planning has been recognized as an important decision-making problem. It seeks to find a simultaneous compromise among the various objective functions of the different divisions. Bi-level programming, a tool for modeling decentralized decisions, consists of the objective(s) of the leader at its first level and that is of the follower at the second level. The decision-maker at each level attempts to optimize his individual objective, which usually depends in part on the variables controlled by the decision-maker at the other levels and their final decisions are executed sequentially where the upper-level decision-maker makes his decision firstly. The research and applications concentrated mainly on bi-level programming (see f. i. [3, 4, 7, 9, 12, 19, 20, 22, 23, 24, 25, 27, 36, 37, 38, 52, 58, 59]).

TOPSIS was first developed by C. L. Hwang and K. Yoon [42] for solving a multiple attribute decision making problem. It is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (*PIS*) and the farthest from the negative ideal solution (*NIS*). The single criterion of the shortest distance from the given goal or the PIS may be not enough to decision makers. In practice , we might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. A similar concept has also been pointed out by M. Zeleny [62], Lia *et al.* [45] extended the concept of TOPSIS to develop a methodology for solving multiple objective decision making (MODM) problems. After the publication of TOPSIS approach [42, 45], the subsequent works in this area of optimization have been numerous (see f. i. [1, 2, 5, 6, 8, 11, 14, 17, 18, 21, 28, 29, 30, 33]).

Abo-Sinna and Abou-El-Enien [5] extend the TOPSIS approach to solve large scale multiple objective decision making problems with block angular structure. Also, they [8] extend the TOPSIS approach to solve large scale multiple objective decision making problems under fuzzy environment. Recently, Baky and Abo-Sinna [21] proposed a TOPSIS algorithm for bi-level multiple objective decision making problems.

In this paper, we extend TOPSIS for solving LS-BL-LVOP, we further extended the concept of TOPSIS [Lia *et al.* (45)] for LS-BL-LVOP.

In the following section, we will give the formulation of LS-BL-LVOP with block angular structure for which the Dantzig-Wolfe decomposition method has been successfully applied. The family of d_p-distance and its normalization is discussed in section 3. The TOPSIS approach is presented in section 4. By use of TOPSIS, we will propose an interactive algorithm for solving LS-BL-LVOP in section 5. We will also give a numerical example in section 6 for the sake of illustration. Finally, concluding remarks will be given in section 7.

2. FORMULATION OF A LS-BL-LVOP:

Consider there are two levels in a hierarchy structure with a first - level decision maker (FLDM) and a second - level decision maker (SLDM). Let the LS-BL-LVOP problem of the following block angular structure :

[FLDM]

$$\underset{X_{I_1}}{Maximize}\; F_{I_1}(X_{I_1},X_{I_2}) = \underset{X_{I_1}}{Maximize}\left(f_{I_1 1}(X_{I_1},X_{I_2}),\ldots,f_{I_1 k_{I_1}}(X_{I_1},X_{I_2})\right)$$

where X_{I_2} *solves second level*

[SLDM]

$$\underset{X_{I_2}}{Maximize}\; F_{I_2}(X_{I_1},X_{I_2}) = \underset{X_{I_2}}{Maximize}\left(f_{I_2 1}(X_{I_1},X_{I_2}),\ldots,f_{I_2 k_{I_2}}(X_{I_1},X_{I_2})\right)$$

subject to ---(1)

$$X \in M = \{X \in R^n : \sum_{j=1}^{q} A_j X_j \le b_o,\quad D_j X_j \le b_j,\quad X_j \ge 0,\ j=1,2,\ldots,q,\ q>1\}$$

where

k : the number of objective functions,

k_{I_1} : the number of objective functions of the FLDM

k_{I_2} : the number of objective functions of the SLDM

n_{I_1} : the number of variables of the FLDM

n_{I_2} : the number of variables of the SLDM

q : the number of subproblems,

m : the number of constraints,

n : the numer of variables,

n_j : the number of variables of the j^{th} subproblem, $j=1,2,\ldots,q$,

m_o : the number of the common constraints represented by $\sum_{j=1}^{q} A_j X_j \le b_o$,

m_j : the number of independent constraints of the j^{th} subproblem represented by $D_j X_j \le b_j$, $j=1,2,\ldots,q$.

A_j : an $(m_o \times n_j)$ coefficient matrix,

D_j : an $(m_j \times n_j)$ coefficient matrix,

b_o : an m_o-dimensional column vector of right-hand sides of the common constraints whose elements are constants,

b_j : an m_j-dimensional column vector of independent constraints right-hand sides whose elements are the constants of the constraints for the j^{th} subproblem, $j=1,2,\ldots,q$,

C_{ij} : an n_j-dimensional row vector for the j^{th} subproblem in the i^{th} objective function,

R : the set of all real numbers,

X : an n-dimensional column vector of variables,

X_j : an n_j -dimensional column vector of variables for the j^{th} subproblem, $j=1,2,\ldots,q$,

X_{I_1} : an n_{I_1} - dimensional column vector of variables of the FLDM,

X_{I_2} : an n_{I_2} - dimensional column vector of variables of the SLDM,

$K = \{1,2,\ldots,k\}$

$N = \{1,2,\ldots,n\}$,

$R^n = \{X=(x_1, x_2,\ldots,x_n)^T : x_i \in R,\ i \in N\}$.

If the objective functions are linear, then the objective function can be written as follows:

$$f_i(X) = \sum_{j=1}^{q} f_{ij} = \sum_{j=1}^{q} C_{ij} X_j,\quad i=1,2,\ldots,k \quad \text{--- (2)}$$

3. SOME BASIC CONCEPTS OF DISTANCE MEASURES:

The compromise programming approach [39, 46, 61, 62] has been developed to perform MODM problem, reducing the set of nondominated solutions. The compromise solutions are those which are the closest by some distance measure to the ideal one.

The point $f_i(X^*) = \sum_{j=1}^{q} f_{ij}(X^*)$ in the criteria space is called the ideal point (reference point). As the measure of "closeness", d_p-metric is used. The d_p-metric defines the distance between two points, $f_i(X) = \sum_{j=1}^{q} f_{ij}(X)$ and $f_i(X^*) = \sum_{j=1}^{q} f_{ij}(X^*)$ (the reference point) in k-dimensional space [50] as:

$$d_p = \left(\sum_{i=1}^{k} w_i^p (f_i^* - f_i)^p\right)^{\frac{1}{p}} = \left(\sum_{i=1}^{k} w_i^p \left(\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}\right)^p\right)^{\frac{1}{p}} \quad \text{--- (3)}$$

where $p \ge 1$.

Unfortunately, because of the incommensurability among objectives, it is impossible to directly use the above distance family. To remove the effects of the incommensurability, we need to normalize the distance family of equation (3) by using the reference point [41, 42] as :

$$d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}}{\sum_{j=1}^{q} f_{ij}^*}\right)^p\right)^{1/p} \quad --- (4)$$

where $p \geq 1$.

To obtain a compromise solution for the Large Scale Vector Optimization problems (LSVOP) of the following form ,

Maximize $[f_1(X), f_2(X), \ldots\ldots, f_k(X)]$
subject to --- (5)

$$X \in M = \{X \in R^n : \sum_{j=1}^{q} A_j X_j \leq b_o,$$
$$D_j X_j \leq b_j,$$
$$X_j \geq 0, j=1,2,\ldots.,q, q>1\}$$

The global criteria method [41] for large scale problems uses the distance family of equation (4) by the ideal solution being the reference point. The problem becomes how to solve the following auxiliary problem :

$$\underset{X\in M}{Minimize}\ d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} f_{ij}(X^*) - \sum_{j=1}^{q} f_{ij}(X)}{\sum_{j=1}^{q} f_{ij}(X^*)}\right)^p\right)^{\frac{1}{p}} --- (6)$$

where X^* is the *PIS* and $p = 1,2,\ldots\ldots,\infty$.
Usually, the solutions based on *PIS* are different from the solutions based on *NIS*. Thus, both $PIS(f^*)$ and $NIS(f^-)$ can be used to normalize the distance family and obtain [18]:

$$d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}}{\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}^-}\right)^p\right)^{1/p} \quad --- (7)$$

where $p \geq 1$.

In this study, we further extended the concept of TOPSIS to obtain a compromise (satisfactory) solution for LS-BL-LVOP problems. Also, in this paper, an algorithm of generating compromise (satisfactory) solutions of LS-BL-LVOP has been presented. It is based on the decomposition algorithm of LSVOP with block angular structure via TOPSIS approach, [5]. This algorithm has few features, (i) it combines both LS-BL-LVOP and TOPSIS approach to obtain TOPSIS's compromise solution of the problem, (ii) it can be efficiently coded. (iii) it was found that the decomposition based method generally met with better results than the traditional simplex-based methods. Especially, the efficiency of the decomposition-based method increased sharply with the scale of the problem. Finally, an illustrative numerical example clarified the various aspects of both the solution concept and the proposed algorithm.

4. TOPSIS for LS-BL-LVOP :

Consider the following LS-BL-LVOP problem with block angular structure:
[FLDM]

$$\underset{X_{I_1}}{Maximize/Minimize}\ F_{I_1}(X_{I_1}, X_{I_2}) = \underset{X_{I_1}}{Maximize/Minimize}\left(f_{I_11}(X_{I_1}, X_{I_2}), \ldots., f_{I_1k_{I_1}}(X_{I_1}, X_{I_2})\right)$$

where X_{I_2} *solves second level*

[SLDM]

$$\underset{X_{I_2}}{Maximize/Minimize}\ F_{I_2}(X_{I_1}, X_{I_2}) = \underset{X_{I_2}}{Maximize/Minimize}\left(f_{I_21}(X_{I_1}, X_{I_2}), \ldots., f_{I_2k_{I_2}}(X_{I_1}, X_{I_2})\right)$$

subject to ---(8)

$X \in M$

where
$\sum_{j=1}^{q} f_{tj}(X)$: Objective Function for Maximization,
$t \in K_1 \sqsubset K$,
$\sum_{j=1}^{q} f_{vj}(X)$: Objective Function for Minimization,
$v \in K_2 \sqsubset K$.

4-1. Phase (I):
Consider the FLDM problem of the LS-BL-LVOP Problem (8):
[FLDM]

$$\underset{X_{I_1}}{Maximize/Minimize}\ F_{I_1}(X_{I_1}, X_{I_2}) = \underset{X_{I_1}}{Maximize/Minimize}\left(f_{I_11}(X_{I_1}, X_{I_2}), \ldots., f_{I_1k_{I_1}}(X_{I_1}, X_{I_2})\right)$$

subject to ---(9)
$X \in M$

where
$\sum_{j=1}^{q} f_{tj}(X)$: Objective Function for Maximization,
$t \in K_1 \sqsubset K$,
$\sum_{j=1}^{q} f_{vj}(X)$: Objective Function for Minimization,
$v \in K_2 \sqsubset K$.
In order to use the distance family of equation (7) to resolve problem (9), we must first find $PIS(f^*)$ and $NIS(f^-)$ which are [18, 45]:

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$$F^{*} = \underset{x \in M^{/}}{Maximize(or\ Minimize)} \sum_{j=1}^{q} F_{tj}(X) \left(or \sum_{j=1}^{q} F_{vj}(X) \right), \quad \forall t (and\ v) --- \quad (10a)$$

$$F^{-} = \underset{x \in M^{/}}{Minimize(or\ Maximize)} \sum_{j=1}^{q} F_{tj}(X) \left(or \sum_{j=1}^{q} F_{vj}(X) \right), \quad \forall t (and\ v) --- \quad (10b)$$

where $K = K_1 \cup K_2$.

$$f^{*^{\mathrm{FLDM}}} = \left(f_1^{*^{FLDM}}, f_2^{*^{FLDM}}, \dots, f_{k_{I_1}}^{*^{FLDM}} \right)$$

and

$$f^{-^{\mathrm{FLDM}}} = \left(f_1^{-^{FLDM}}, f_2^{-^{FLDM}}, \dots \dots, f_{k_{I_1}}^{-^{FLDM}} \right)$$

are the individual positive (negative) ideal solutions for the FLDM.

Using the *PIS* and the *NIS* for the FLDM, we obtain the following distance functions from them, respectively:

$$d_P^{PIS^{\mathrm{FLDM}}} = \left(\sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{*^{\mathrm{FLDM}}} - \sum_{j=1}^q f_{tj}^{\mathrm{FLDM}}(X)}{\sum_{j=1}^q f_{tj}^{*^{\mathrm{FLDM}}} - \sum_{j=1}^q f_{tj}^{-^{FLDM}}} \right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{FLDM}(X) - \sum_{j=1}^q f_{vj}^{*^{FLDM}}}{\sum_{j=1}^q f_{vj}^{-^{FLDM}} - \sum_{j=1}^q f_{vj}^{*^{FLDM}}} \right)^p \right)^{1/p} - (11a)$$

and

$$d_P^{NIS^{\mathrm{FLDM}}} = \left(\sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{FLDM}(X) - \sum_{j=1}^q f_{tj}^{-^{FLDM}}}{\sum_{j=1}^q f_{tj}^{*^{FLDM}} - \sum_{j=1}^q f_{tj}^{-^{FLDM}}} \right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{-^{FLDM}} - \sum_{j=1}^q f_{vj}^{FLDM}(X)}{\sum_{j=1}^q f_{vj}^{-^{FLDM}} - \sum_{j=1}^q f_{vj}^{*^{FLDM}}} \right)^p \right)^{1/p} --- (11b)$$

where $w_i = 1,2, \dots, k$, are the relative importance (weighs) of objectives, and $p = 1,2, \dots, \infty$.

In order to obtain a compromise solution for the FLDM, we transfer the FLDM of problem (9) into the following bi-objective problem with two commensurable (but often conflicting) objectives [18, 45]:

$$Minimize\ d_p^{PIS^{FLDM}}(X),\ Maximize\ d_p^{NIS^{FLDM}}(X)$$

subject to $\qquad ---(12)$

$X \in M$

where $p = 1,2, \dots, \infty$.

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_1(X)$ and $\mu_2(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the *PIS* for $\mu_1(X)$and assign a larger degree to the one with farther distance from *NIS* for $\mu_2(X)$. Therefore, as shown in figure (1), $\mu_1(X) \equiv \mu_{d_p^{PIS^{FLDM}}}(X)$ and $\mu_2(X) \equiv \mu_{d_p^{NIS^{FLDM}}}(X)$ can be obtained as the following (see [26, 43, 44, 53, 55, 63]):

$$\mu_1(X) = \begin{cases} 1, & if\ d_p^{PIS}(X) < \left(d_p^{PIS}\right)^*, \\ 1 - \dfrac{d_p^{PIS}(X) - \left(d_p^{PIS}\right)^*}{\left(d_p^{PIS}\right)^- - \left(d_p^{PIS}\right)^*}, & if\ \left(d_p^{PIS}\right)^- \geq d_p^{PIS} \geq \left(d_p^{PIS}\right)^* \\ 0, & if\ d_p^{PIS}(X) > \left(d_p^{PIS}\right)^-, \end{cases} --- (13a)$$

$$\mu_2(X) = \begin{cases} 1, & if\ d_p^{NIS}(X) > \left(d_p^{NIS}\right)^*, \\ 1 - \dfrac{d_p^{NIS}(X) - \left(d_p^{NIS}\right)^-}{\left(d_p^{NIS}\right)^* - \left(d_p^{NIS}\right)^-}, & if\ \left(d_p^{NIS}\right)^- \leq d_p^{NIS} \leq \left(d_p^{NIS}\right)^*, ---(13b) \\ 0, & if\ d_p^{NIS}(X) < \left(d_p^{NIS}\right)^-, \end{cases}$$

where

$$\left(d_p^{PIS}\right)^* = \underset{x \in M^{/}}{Minimize}\, d_p^{PIS}(X) \text{ and the solution is } X^{PIS},$$
$$\left(d_p^{NIS}\right)^* = \underset{x \in M^{/}}{Maximize}\, d_p^{NIS}(X) \text{ and the solution is } X^{NIS},$$
$$\left(d_p^{PIS}\right)^- = d_p^{PIS}(X^{NIS}) \text{ and } \left(d_p^{NIS}\right)^- = d_p^{NIS}(X^{PIS}).$$

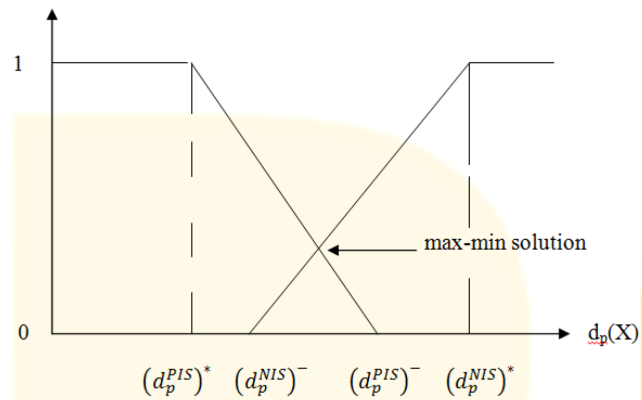

Figure 1. The membership functions of $\mu_1(X)$ and $\mu_2(X)$

Now, by applying the max-min decision model which is proposed by R. E. Bellman and L. A. Zadeh [26] and extended by H. –J. Zimmermann [63], we can resolve problem (12). The satisfying decision of the FLDM of the LS-BL-LVOP Problem, $X^{*\text{FLDM}} = (X_{I_1}^{*\text{FLDM}}, X_{I_2}^{*\text{FLDM}})$, may be obtained by solving the following model:

$$\mu_D\left(X^{*\text{FLDM}}\right) = \underset{X \in M}{Maximize} \left\{Min.\left(\mu_1(X),\ \mu_2(X)\right)\right\} \qquad (14)$$

Finally, if $\delta^{FLDM} = Minimize\ (\mu_1(X),\ \ \mu_2(X))$, the model (14) is equivalent to the form of Tchebycheff model (see [32]), which is equivalent to the following model:

$Maximize\ \delta^{FLDM}, ---(15a)$

subject to

$\mu_1(X) \geq \delta^{FLDM} --- (15b),$

$\mu_2(X) \geq \delta^{FLDM} --- (15c)$

$X \in M, \quad \delta^{FLDM} \in [0,1] --- (15d)$

where δ^{FLDM} is the satisfactory level for both criteria of the shortest distance from the *PIS* and the farthest distance from the *NIS*. It is well known that if the optimal solution of (15) is the vector $\left(\delta^{*FLDM}, X^{*\text{FLDM}}\right)$, then $X^{*\text{FLDM}}$is a nondominated solution [41,60] of (12) and a satisfactory solution [46] of the FLDM problem (9).

The basic concept of the bi-level programming technique is that the FLDM sets his/her goals and/or decisions with possible tolerances which are described by membership functions of fuzzy set theory. According to this concept, let τ_i^L and $\tau_i^R, i = 1,2, \dots, n_{I_1}$ be the maximum acceptable negative and positive tolerance (relaxation) values on the decision vector considered by the FLDM, $X_{I_1}^{*FLDM} = \left(x_{I_1 1}^{*FLDM}, x_{I_1 2}^{*FLDM}, \dots, x_{I_1 n_{I_1}}^{*FLDM}\right)$. The tolerances give the SLDM an extent feasible region to search for the satisfactory solution. If the feasible region is empty, the negative and positive tolerances must be increased to give the *SLDM* an extent feasible region to search for the satisfactory solution, [11, 51, 57]. The linear membership functions (Figure 2) for each of the n_{I_1}components of the decision vector $\left(x_{I_1 1}^{*FLDM}, x_{I_1 2}^{*FLDM}, \dots, x_{I_1 n_{I_1}}^{*FLDM}\right)$ controlled by the FLDM can be formulated as:

$$\mu_{I_1 i}(x_{I_1 i}) = \begin{cases} \dfrac{x_{I_1 i} - \left(X_{I_1 i}^{*FLDM} - \tau_i^L\right)}{\tau_i^L}, & if\ x_{I_1 i}^{*FLDM} - \tau_i^L \leq x_{I_1 i} \leq x_{I_1 i}^{*FLDM} \\ \dfrac{\left(X_{I_1 i}^{*FLDM} + \tau_i^R\right) - x_{I_1 i}}{\tau_i^R}, & if\ x_{I_1 i}^{*FLDM} \leq x_{I_1 i} \leq x_{I_1 i}^{*FLDM} + \tau_i^R, \quad i = 1,2, \dots, n_{I_1}, \\ 0, & if\ otherwise, \end{cases} \qquad --- (16)$$

It may be noted that, the decision maker may desire to shift the range of $x_{I_1 i}$. Following Pramanik & Roy [51] and Sinha [57], this shift can be achieved.

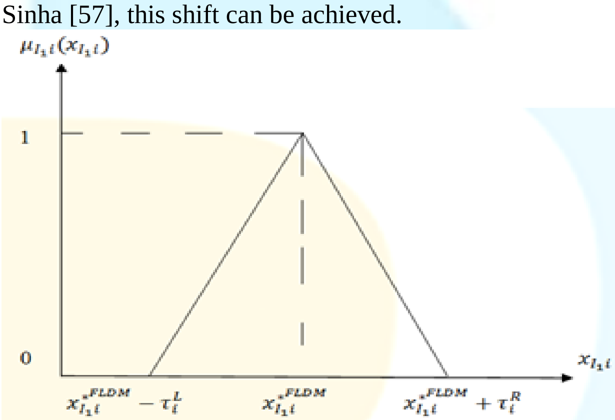

Figure 2: The membership function of the decision variable $x_{I_1 i}$

4-2. Phase (II):

The SLDM problem can be written as follows:

[SLDM]

$$\underset{X_{I_2}}{Maximize}\ F_{I_2}(X_{I_1}, X_{I_2}) = \underset{X_{I_2}}{Maximize}\left(f_{I_2 1}(X_{I_1}, X_{I_2}), \dots, f_{I_2 k_{I_2}}(X_{I_1}, X_{I_2})\right)$$

$subject\ to \qquad --(17)$

$X \in M$

where

$\sum_{j=1}^{q} f_{tj}\ (X)$: Objective Function for Maximization,

$t \in K_1 \sqsubset K,$

$\sum_{j=1}^{q} f_{vj}\ (X)$: Objective Function for Minimization,

$v \in K_2 \sqsubset K.$

In order to use the distance family of equation (7) to resolve problem (17), we must first find $PIS(f^*)$ and $NIS(f^-)$which are [18, 45]:

$f^{*SLDM} = \underset{x \in M}{Maximize(or\ Minimize)} \sum_{j=1}^{q} f_{tj}(X) \left(or \sum_{j=1}^{q} f_{vj}(X)\right)$, $\forall t(and\ v)$ − − − (18 − a)

$f^{-SLDM} = \underset{x \in M}{Minimize(or\ Maximize)} \sum_{j=1}^{q} f_{tj}(X) \left(or \sum_{j=1}^{q} f_{vj}(X)\right)$, $\forall t(and\ v)$ − − − (18 − b)

where $K = K_1 \cup K_2$,

$f^{*SLDM} = \left(f_1^{*SLDM}, f_2^{*SLDM}, \dots, f_{k_{I_2}}^{*SLDM}\right)$ and $f^{-SLDM} = \left(f_1^{-SLDM}, f_2^{-SLDM}, \dots\dots, f_{k_{I_2}}^{-SLDM}\right)$ are the individual positive (negative) ideal solutions for the SLDM.

In order to obtain a compromise (satisfactory) solution to the LS-BL-LVOP using TOPSIS approach, the distance family of (7) to represent the distance function from the positive ideal solution, $d_P^{PIS^{BL}}$, and the distance function from the negative ideal solution, $d_P^{NIS^{BL}}$, can be proposed, in this paper, for the objectives of the FLDM and the SLDM as follows:

$$d_P^{PIS^{BL}} = \left(\sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{*FLDM} - \sum_{j=1}^q f_{tj}^{FLDM}(X)}{\sum_{j=1}^q f_{tj}^{*FLDM} - \sum_{j=1}^q f_{tj}^{-FLDM}} \right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{FLDM}(X) - \sum_{j=1}^q f_{vj}^{*FLDM}}{\sum_{j=1}^q f_{vj}^{-FLDM} - \sum_{j=1}^q f_{vj}^{*FLDM}} \right)^p + \sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{*SLDM} - \sum_{j=1}^q f_{tj}^{SLDM}(X)}{\sum_{j=1}^q f_{tj}^{*SLDM} - \sum_{j=1}^q f_{tj}^{-SLDM}} \right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{SLDM}(X) - \sum_{j=1}^q f_{vj}^{*SLDM}}{\sum_{j=1}^q f_{vj}^{-SLDM} - \sum_{j=1}^q f_{vj}^{*SLDM}} \right)^p \right)^{1/p} \quad ---(19a)$$

And

$$d_P^{NIS^{BL}} = \left(\sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{FLDM}(X) - \sum_{j=1}^q f_{tj}^{-FLDM}}{\sum_{j=1}^q f_{tj}^{*FLDM} - \sum_{j=1}^q f_{tj}^{-FLDM}} \right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{-FLDM} - \sum_{j=1}^q f_{vj}^{FLDM}(X)}{\sum_{j=1}^q f_{vj}^{-FLDM} - \sum_{j=1}^q f_{vj}^{*FLDM}} \right)^p + \sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^q f_{tj}^{SLDM}(X) - \sum_{j=1}^q f_{tj}^{-SLDM}}{\sum_{j=1}^q f_{tj}^{*SLDM} - \sum_{j=1}^q f_{tj}^{-SLDM}} \right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q f_{vj}^{-SLDM} - \sum_{j=1}^q f_{vj}^{SLDM}(X)}{\sum_{j=1}^q f_{vj}^{-SLDM} - \sum_{j=1}^q f_{vj}^{*SLDM}} \right)^p \right)^{1/p} \quad ---(19b)$$

where $w_i = 1,2,\dots,k$, are the relative importance (weighs) of objectives, and $p = 1,2,\dots\dots,\infty$.

In order to obtain a compromise solution, we transfer problem (8) into the following bi-objective problem with two commensurable (but often conflicting) objectives [18, 45]:

$$Minimize\ d_p^{PIS^{BL}}(X),\ Maximize\ d_p^{NIS^{BL}}(X)$$
subject to $\qquad ---(20)$
$X \in M$

where $p = 1,2,\dots\dots,\infty$.

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_3(X)$ and $\mu_4(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the *PIS* for $\mu_3(X)$ and assign a larger degree to the one with farther distance from *NIS* for $\mu_4(X)$. Therefore, as shown in figure (3), $\mu_3(X) \equiv \mu_{d_p^{PIS^{BL}}}(X)$ and $\mu_4(X) \equiv \mu_{d_p^{NIS^{BL}}}(X)$ can be obtained as the following (see [26, 43, 44, 53, 55, 63]):

$$\mu_3(X) = \begin{cases} 1, & if\ d_p^{PIS^{BL}}(X) < \left(d_p^{PIS^{BL}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{BL}}(X) - \left(d_p^{PIS^{BL}}\right)^*}{\left(d_p^{PIS^{BL}}\right)^/ - \left(d_p^{PIS^{BL}}\right)^*}, & if\ \left(d_p^{PIS^{BL}}\right)^- \geq d_p^{PIS^{BL}}(X) \geq \left(d_p^{PIS^{BL}}\right)^*, \\ 0, & if\ d_p^{PIS^{BL}}(X) > \left(d_p^{PIS^{BL}}\right)^-, \end{cases} \quad ---(21a)$$

$$\mu_4(X)=\begin{cases}1, & if\, d_p^{NIS^{BL}}(X) > \left(d_p^{NIS^{BL}}\right)^*,\\ 1-\dfrac{\left(d_p^{NIS^{BL}}\right)^* - d_p^{NIS^{BL}}(X)}{\left(d_p^{NIS^{BL}}\right)^* - \left(d_p^{NIS^{BL}}\right)^-}, & if\, \left(d_p^{NIS^{BL}}\right)^- \le d_p^{NIS^{BL}}(X) \le \left(d_p^{NIS^{BL}}\right)^*, ---(21b)\\ 0, & if\, d_p^{NIS^{BL}}(X) < \left(d_p^{NIS^{BL}}\right)^-,\end{cases}$$

where

$$\left(d_p^{PIS^{BL}}\right)^* = \underset{x\in M}{Minimize}\; d_p^{PIS^{BL}}(X)\ \ and\ the\ solution\ is\ X^{PIS^{BL}},$$
$$\left(d_p^{NIS^{BL}}\right)^* = \underset{x\in M}{Maximize}\; d_p^{NIS^{BL}}(X)\ \ and\ the\ solution\ is\ X^{NIS^{BL}},$$
$$\left(d_p^{PIS^{BL}}\right)^- = d_p^{PIS^{BL}}\left(X^{NIS^{BL}}\right) and \left(d_p^{NIS^{BL}}\right)^- = d_p^{NIS^{BL}}\left(X^{PIS^{BL}}\right).$$

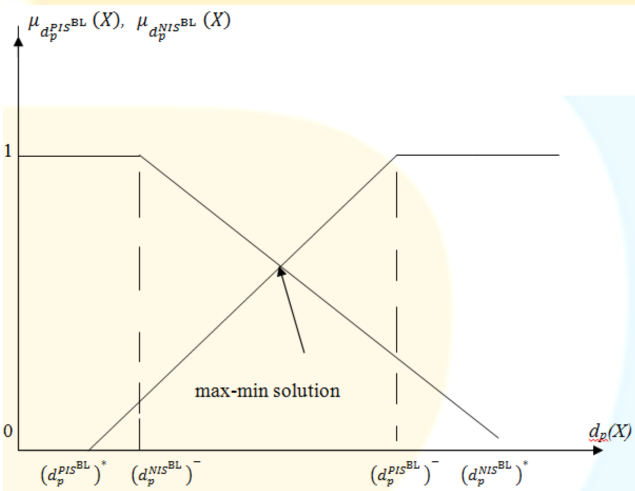

Figure 3: The membership functions of $\mu_{d_p^{PIS^{BL}}}(X)$ and $\mu_{d_p^{NIS^{BL}}}(X)$

Now, by applying the max-min decision model which is proposed by R. E. Bellman and L. A. Zadeh [26] and extended by H. –J. Zimmermann [63], we can resolve problem (20). The satisfactory solution of the LS-BL-LMOP Problem, $X^{*^{BL}}$, may be obtained by solving the following model:

$$\mu_D\left(X^{*^{BL}}\right) = \underset{x\in M}{Maximize}\ \{Min.(\mu_3(X),\quad \mu_4(X))\} ---(22)$$

Finally, if $\delta^{BL} = Minimize\ (\mu_3(X),\ \mu_4(X))$, the model (22) is equivalent to the form of Tchebycheff model (see [32]), which is equivalent to the following model:

$Maximize\ \delta^{BL}, ---(23a)$
subject to
$\mu_3(X) \ge \delta^{BL}, ---(23b)$
$\mu_4(X) \ge \delta^{BL}, ---(23c)$

$$\frac{x_{I_1 i} - \left(X_{I_1 i}^{*FLDM} - \tau_i^L\right)}{\tau_i^L} \ge \delta^{BL}, i = 1,2,\dots,n_{I_1} --- (23d)$$

$$\frac{\left(X_{I_1 i}^{*FLDM} + \tau_i^R\right) - x_{I_1 i}}{\tau_i^R} \ge \delta^{BL}, i = 1,2,\dots,n_{I_1} --- (23e)$$

$x \in M,\quad \delta^{BL} \in [0,1], ---(23f)$

where δ^{BL} is the satisfactory level for both criteria of the shortest distance from the *PIS* and the farthest distance from the *NIS*. It is well known that if the optimal solution of (23) is the vector $\left(\delta^{*^{BL}}, X^{*^{BL}}\right)$, then $X^{*^{BL}}$ is a nondominated solution of (20) and a satisfactory solution for the LS-BL-LMOP problem.

5. THE INTERACTIVE ALGORITHM OF TOPSIS FOR SOLVING LS-BL-LVOP:

Thus, we can introduce the following interactive algorithm to gernerate a set of satisfactory solutions for the LS-BL-LVOP:

The algorithm (Alg-I):

Phase (I):

Step 1. Construct the *PIS* payoff table of problem (9) by using the decomposition algorithm [31], and obtain $f^{*^{FLDM}} = \left(f_1^{*FLDM}, f_2^{*FLDM}, \dots, f_{k_{I_1}}^{*FLDM}\right)$ the individual positive ideal solutions.

Step 2. Construct the *NIS* payoff table of problem (9) by using the decomposition algorithm, and obtain $f^{-^{FLDM}} = (f_1^{-FLDM}, f_2^{-FLDM}, \dots, f_{k_{I_1}}^{-FLDM})$, the individual negative ideal solutions.

Step 3. Use equations (10 & 11) and the above steps (1 & 2) to construct $d_p^{PIS^{FLDM}}$ and $d_p^{NIS^{FLDM}}$.

Step 4. Transform problem (9) to the form of problem (12).

Step 5. (I) Ask the FLDM to select
p= p*∈ {1, 2, …, ∞ },
(II) Ask the FLDM to select $w_i = w_i{}^*$, $i = 1,2,\dots,k_{I_1}$, where $\sum_{i=1}^{k_{I_1}} w_i = 1$,

Step 6. Use steps (3 & 5) to compute $d_p^{PIS^{FLDM}}$ and $d_p^{NIS^{FLDM}}$.

Step 7. Construct the payoff table of problem (12):
At $p = 1$, use the decomposition algorithm [31].
At $p \ge 2$, use the generalized reduced gradient method, [47, 48], and obtain:

$$d_p^{-FLDM} = \left(\left(d_p^{PIS^{FLDM}}\right)^-, \left(d_p^{NIS^{FLDM}}\right)^-\right),$$
$$d_p^{*FLDM} = \left(\left(d_p^{PIS^{FLDM}}\right)^*, \left(d_p^{NIS^{FLDM}}\right)^*\right).$$

Step 8. Construct problem (15) by using the membership functions (13).

Step 9. Solve problem (15) to obtain $\left(\delta^{*FLDM}, X^{*^{FLDM}}\right)$.

Step10. Ask the FLDM to select the maximum negative and positive tolerance values τ_i^L and τ_i^R, $i = 1,2,\dots,n_{I_1}$ on the decision vector $X_{I_1}^{*FLDM} = \left(x_{I_11}^{*FLDM}, x_{I_12}^{*FLDM}, \dots, x_{I_1n_{I_1}}^{*FLDM}\right)$,

Phase (II):

Step 11. Construct the *PIS* payoff table of problem (17) by using the decomposition algorithm [31], and obtain $f^{*SLDM} = \left(f_1^{*SLDM}, f_2^{*SLDM}, \dots, f_{k_{I_2}}^{*SLDM}\right)$ the individual positive ideal solutions.

Step 12. Construct the NIS payoff table of problem (17) by using the decomposition algorithm, and obtain $f^{-SLDM} = (f_1^{-SLDM}, f_2^{-SLDM}, \dots, f_{k_{I_2}}^{-SLDM})$ the individual negative ideal solutions.

Step 13. Use equations (18 & 19) and the above steps (11 & 12) to construct $d_p^{PIS^{BL}}$ and $d_p^{NIS^{BL}}$.

Step 14. Transform problem (8) to the form of problem (20).

Step 15. Ask the FLDM to select $w_i = w_i^*$, $i = 1,2,\dots,k$, where $\sum_{i=1}^{k} w_i = 1$,

Step 16. Use steps (5-I , 13 , 15) to compute $d_p^{PIS^{BL}}$ and $d_p^{NIS^{BL}}$.

Step 17. Construct the payoff table of problem (20): At p=1, use the decomposition algorithm [31], At p≥2, use the generalized reduced gradient method, [47, 48], and obtain:

$$d_p^{-BL} = \left(\left(d_p^{PIS^{BL}}\right)^-, \left(d_p^{NIS^{BL}}\right)^-\right),$$

$$d_p^{*BL} = \left(\left(d_p^{PIS^{BL}}\right)^*, \left(d_p^{NIS^{BL}}\right)^*\right).$$

Step 18. Use equations (16 and 21) to construct problem (23).

Step 19. Solve problem (23) to obtain $\left(\delta^{*BL}, X^{*BL}\right)$.

Step 20. If the DM is satisfied with the current solution , go to step 21. Otherwise, go to step 5.

Step 21. Stop.

6. AN ILLUSTRATIVE NUMERICAL EXAMPLE:

Consider the following LS-BL-LVOP problem with block angular structure:

[FLDM]

$$\underset{x_1}{Maximize}\; f_{11}(X) = 2x_1 + 4x_2 --- (24a)$$

$$\underset{x_1}{Minimize}\; f_{12}(X) = x_1 - 2x_2 --- (24b)$$

where x_1 solves the second level

[SLDM]

$$\underset{x_2}{Maximize}\; f_{21}(X) = 2x_1 + 5x_2 --- (24c)$$

$$\underset{x_2}{Minimize}\; f_{22}(X) = 2x_1 - 4x_2 --- (24d)$$

subject to

$$x_1 + x_2 \le 5, --- (24e)$$
$$x_1 \le 2, --- (24f)$$
$$x_2 \le 4, --- (24g)$$
$$x_1, x_2 \ge 0 --- (24h)$$

Solution:

Obtain *PIS* and *NIS* payoff tables for the FLDM of the LS-BL-LVOP Problem (24).

Table (1) : *PIS* payoff table for the FLDM of problem (24)

	f_{11}	f_{12}	x_1	x_2
$\underset{x_1}{Maximize}\, f_{11}(X)$	*18**	*-7*	*1*	*4*
$\underset{x_1}{Minimize}\, f_{12}(X)$	*16*	*-8**	*0*	*4*

PIS: f^{*FLDM}=(18 , -8)

Table (2) : *NIS* payoff table for the FLDM of problem (24)

	f_{11}	f_{12}	x_1	x_2
$\underset{x_1}{Minimize}\, f_{11}(X)$	0^-	*0*	*0*	*0*
$\underset{x_1}{Maximize}\, f_{12}(X)$	4	2^-	*2*	*0*

NIS: f^{-FLDM}=(0 , 2)

Next, compute equation (11) and obtain the following equations:

$$d_P^{PIS^{FLDM}} = \left[w_1^p\left(\frac{18 - f_{11}(X)}{18 - 0}\right)^p + w_2^p\left(\frac{f_{12}(x) - (-8)}{2 - (-8)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{FLDM}} = \left[w_1^p\left(\frac{f_{11}(X) - 0}{18 - 0}\right)^p + w_2^p\left(\frac{2 - f_{12}(x)}{2 - (-8)}\right)^p\right]^{1/p}$$

Thus, problem (12) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and $p=2$,

Table (3) : *PIS* payoff table of problem (12), when *p=2*.

	$d_2^{PIS^{FLDM}}$	$d_2^{NIS^{FLDM}}$	x_1	x_2
$Min.\, d_2^{PIS^{FLDM}}$	0.0365*	0.5677^-	*0.5339*	4
$Max.\, d_2^{NIS^{FLDM}}$	0.05^-	0.6592*	*1*	4

d_2^{*FLDM}=(0.0365 , 0.6592) , d_2^{-FLDM}=(0.05 , 0.6577).

Now, it is easy to compute (15) :

$$Maximize\; \delta^{FLDM}$$

subject to

$$x_1 + x_2 \le 5,\; x_1 \le 2,\; x_2 \le 4,\; x_1, x_2 \ge 0,$$

$$\left(\frac{d_2^{PIS^{FLDM}}(X) - 0.0365}{0.0135}\right) \ge \delta^{FLDM},$$

$$\left(\frac{0.6592 - d_2^{NIS^{FLDM}}(X)}{0.0015}\right) \ge \delta^{FLDM},$$

$$\delta^{FLDM} \in [0,1] \;.$$

The maximum "satisfactory level" ($\delta^{\text{FLDM}}=1$) is achieved for the solution $X_1^{*FLDM}=1$, $X_2^{*FLDM}=1$ and $(f_{11}, f_{12}) = (18, -8)$. Let the FLDM decide $X_1^{*FLDM}=1$ with positive tolerance $\tau^R = 0.5$ (one sided membership function [21, 52, 58]).
Obtain *PIS* and *NIS* payoff tables for the SLDM of the LS-BL-LMOP Problem (24).

Table (4) : *PIS* payoff table for the SLDM of problem (24)

	f_{21}	f_{22}	x_1	x_2
$\underset{x_2}{Maximize}\, f_{21}(X)$	23^*	-10	1	4
$\underset{x_2}{Minimize}\, f_{22}(X)$	20	-12^*	0	4

PIS: $f^{*\text{SLDM}}$=(23 , -12)

Table (5) : *NIS* payoff table for the SLDM of problem (24)

	f_{21}	f_{22}	x_1	x_2
$\underset{x_2}{Minimize}\, f_{21}(X)$	0^-	0	0	0
$\underset{x_2}{Maximize}\, f_{22}(X)$	6	4^-	2	0

NIS: $f^{-\text{SLDM}}$ =(0 , 4)
Next, compute equation (19) and obtain the following equations:

$$d_P^{PIS^{\text{BL}}} = \left[w_1^p \left(\frac{18 - f_{11}(X)}{18 - 0}\right)^p + w_2^p \left(\frac{f_{12}(x) - (-8)}{2 - (-8)}\right)^p + w_3^p \left(\frac{23 - f_{21}(X)}{23 - 0}\right)^p + w_4^p \left(\frac{f_{22}(X) - (-12)}{4 - (-12)}\right)^p \right]^{1/p}$$

$$d_P^{NIS^{\text{BL}}} = \left[w_1^p \left(\frac{f_{11}(X) - 0}{18 - 0}\right)^p + w_2^p \left(\frac{2 - f_{12}(x)}{2 - (-8)}\right)^p + w_3^p \left(\frac{f_{21}(X) - 0}{23 - 0}\right)^p + w_4^p \left(\frac{4 - f_{22}(X)}{4 - (-12)}\right)^p \right]^{1/p}$$

Thus, problem (20) is obtained. In order to get numerical solutions, assume that $w_1^p=w_2^p=w_3^p=w_4^p=0.25$ and $p=2$,

Table (6) : *PIS* payoff table of problem (20), when $p=2$.

	$d_2^{PIS^{\text{BL}}}$	$d_2^{NIS^{\text{BL}}}$	x_1	x_2
$Min.\, d_2^{PIS^{\text{BL}}}$	0.0292^*	0.4709^-	0.5340	4
$Max.\, d_2^{NIS^{\text{BL}}}$	0.04^-	0.4727^*	1	4

d_2^{*BL}=(0.0292 , 0.4727) , d_2^{-BL}=(0.04 , 0.4709).
Now, it is easy to compute (23) :
$Maximize\ \delta^{\text{BL}}$
subject to

$$x_1 + x_2 \le 5,\ x_1 \le 2,\ x_2 \le 4,$$
$$x_1, x_2 \ge 0,$$

$$\left(\frac{d_2^{PIS^{\text{BL}}}(X) - 0.0292}{0.04 - 0.0292}\right) \ge \delta^{\text{BL}},$$

$$\left(\frac{0.4727 - d_2^{NIS^{\text{BL}}}(X)}{0.4727 - 0.4709}\right) \ge \delta^{\text{BL}},$$

$$\left(\frac{(1 + 0.5) - x_1}{0.5}\right) \ge \delta^{\text{BL}}$$

$$\delta^{\text{BL}} \in [0,1] \ .$$

The maximum "satisfactory level" ($\delta^{\text{BL}}=1$) is achieved for the solution $X_1^{*BL}=1$, $X_2^{*BL}=1$.

7. CONCLUSION

In this paper, a TOPSIS approach has been extended to solve LS-BL-LVOP . The LS-BL-LVOP using TOPSIS approach provides an effective way to find the compromise (satisfactory) solution of such problems. In order to obtain a compromise (satisfactory) solution to the LS-BL-LVOP using the proposed TOPSIS approach, a modified formulas for the distance function from the PIS and the distance function from the NIS are proposed and modeled to include all objective functions of both the first and the second levels. Thus, the bi-objective problem is obtained which can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS, compromise solution by a second–order compromise. The max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the *PIS* and the longest distance from the *NIS*). An interactive TOPSIS algorithm for solving these problems are also proposed. An illustrative numerical example is given to demonstrate the proposed TOPSIS approach and the interactive algorithm.

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