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# The Impact of Order Quantity on the Returns Policy in Supply Chain Coordination

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**Abstract-** *Increasing orders is a major goal for manufacturers. For this, the coordination mechanism and returns policy are often viewed as useful tools. Accordingly, this paper analyzes the impact of order quantity on the returns policy in a supply chain. We investigate the effect of order decisions in the individual channel versus the coordinating channel. Moreover, we compare the difference between the returns policy and no-returns policy on the ordering decision. The results indicate that a retailer's optimal order quantity in the coordinating channel is larger than that in the individual channel. Furthermore, when the returns policy is ignored, we find that the retailer's optimal order quantity in the coordinating channel will be twice as large as that in the individual channel. In addition, the retailer does not always orders higher quantities when the manufacturer provides a returns policy. The higher quantities only exist in the two parties' self-interested manner. However, in the joint model, the optimal order quantity of the retailer is the same before and after return. These results conclude that the returns policy is relevant and positive factor in the individual model, whereas it becomes irrelevant in the coordinated model.*

**Keywords-** *Supply Chain Coordination; Returns Policy; Demand Uncertainty; Order Quantity*

## 1. INTRODUCTION

A returns policy is also often used as an important competitive advantage for manufacturers to increase order quantity. Generally, the manufacturer wishes that the retailer can order more products in order to increase the expected profit. Thus, the manufacturer may offer some favorable terms (e.g., returns policy or quantity discount) to encourage the retailer to increase the order quantity. That is, the retailer can return unsold products to the manufacturer at a certain unit price which is lower than the wholesale price or the manufacturer offers a lower price when the retailer purchases large orders.

Pasternack (1985) first stated that the returns policy could achieve channel coordination if manufacturers set an appropriate coordinated pricing. Emmons and Gilbert (1998) demonstrated that a returns policy can enhance the total expected profit in a supply chain when demand distribution was price-dependent. Subsequently, Mantrala and Raman (1999) extended the work of Emmons and Gilbert (1998) to analyze how different levels of demand variability affect the retailer's optimal order quantity for the manufacturer's wholesale and buyback prices. After that, Lau and Lau (2002) investigated the impact of reducing demand uncertainty in the manufacturer and retailer channel for single period products. Later, Hua et al. (2006) extended the research of Lau and Lau (2002) to consider a cooperative game between a manufacturer and a retailer as well as to verify that the outcome was consistent with theirs. Recently, Zhou and Li (2007) modeled a supply chain contract with one manufacturer and one retailer to investigate the influence of different ordering strategies and the expected profits of the manufacturer-retailer in the whole supply chain. They suggested that if the manufacturer is willing to raise the unit return price to the retailer, the expected profit of the whole supply chain will be enhanced. Brown et al. (2007) compared the distributor's expected profit and order quantity associated with the pooled and non-pooled returns policies. They pointed out that the distributor can get a higher profit under the pooled returns policy. However, they found that the distributor's optimal order quantity with the pooled returns policy is smaller than that with non-pooled returns policy. Yao et al. (2008) analyzed the impact of price-sensitivity on the returns policy in a supply chain. Wang and Zipkin (2009) used a buy-back contract to investigate how the behavior of decisions makers to affect the performance of a two-stage supplier-retailer system. Xiao et al. (2010) investigated the coordination of a supply chain facing consumer return. Chen (2011) considered the returns with wholesale-price-discount contract in a newsvendor problem. Li et al. (2013) examined the relationships among the returns policy, product quality, and pricing strategy in online selling.

Hua et al. (2006) provided a meaning guideline regarding the importance of cooperation since they emphasized that cooperative supply chain could enhance the overall coordinating channel profit. However, in contrast to our models, they did not consider the returns policy. Since the returns policy is also a popular mechanism to add order quantities for the purpose of profit improved. Accordingly, we extend their research to address a returns policy under demand uncertainty. One of the main motivations of this study is to understand whether a returns policy can actually increase the retailer's order quantity to improve both the manufacturer's and retailer's profits. Generally, the retailer may either consider his own profit in view of self-interest to execute the optimal ordering decision or coordinate with the manufacturer in view of joint interest to make the optimal decision. Therefore, we further compare the difference of the two situations of self-interest and joint interest regarding the order policy. In this paper, we consider a supply chain system consisting of one manufacturer and one retailer. Our objective is to provide a more complete and deeper insight to understand how the returns policy affects the order decision for both the manufacturer and retailer.

Intuitively, a manufacturer provides a returns policy that will entice the retailer to order greater quantities under uncertain demand. However, our results show that the retailer is not always attracted to buy more quantities when the manufacturer offers a returns policy. Under the non-coordinating model, the optimal order quantity with returns policy is more than that without returns policy. However, under the coordinating model, the retailer's optimal order quantity in the return scenario will be the same as that in the no-returns scenario. Moreover, we note that when the manufacturer and retailer take the coordinating strategy, the order quantity will increase. This reveals that a coordination mechanism can certainly increase the order quantity.

The rest of this paper is organized as follows. Notations and assumptions for deriving the proposed model are given in Section 2. Section 3 discusses the models. Section 4 presents the analysis and discussion. Finally, conclusions are presented in Section 5.

## 2. NOTATIONS AND ASSUMPTIONS

The notations and assumptions used in the proposed model are listed below.

### 2.1 Notations:

| | |
|---|---|
| $p$ | retail price per unit |
| $c$ | production cost per unit |
| $\beta$ | shortage cost per unit |
| $s$ | return price per unit |
| $w$ | wholesale price per unit |
| $w_{s=0}$ | wholesale price per unit when $s=0$ |
| $Q$ | order quantity in the individual model |
| $Q_j$ | order quantity in the joint model |
| $Q_{s=0}$ | order quantity in the individual model when $s=0$ |
| $Q_{j,s=0}$ | order quantity in the joint model when $s=0$ |
| $D$ | market demand |
| $f(\cdot)$ | probability density function of $D$ |
| $F(\cdot)$ | cumulative distribution function of $D$ |
| $\mu$ | mean of demand uncertainty |
| $\sigma$ | standard deviation of demand uncertainty |
| $E(\pi_r)$ | retailer's expected profit |
| $E(\pi_m)$ | manufacturer's expected profit |
| $E(\pi_{r+m})$ | sum of $E(\pi_r)$ and $E(\pi_m)$ |
| $E(\pi_j)$ | the expected profits in the joint model |
| $E(\pi_{r,s=0})$ | retailer's expected profit when $s=0$ |
| $E(\pi_{m,s=0})$ | manufacturer's expected profit when $s=0$ |

### 2.2 Assumptions:

1. Demand uncertainty is measured by coefficient of variation and follows a uniform distribution.
2. For simplicity, we assume that the retailer price is fixed.
3. We assume the retailer only orders one time, so when the actual market demand exceeds the stock, the retailer will yield goodwill loss; On the contrary, the retailer will return the residual products to the manufacturer.
4. The unit production cost and return price are constant.
5. The return price is lower than the wholesale price.

## 3. THE MODEL

In this section, we first propose the individual profit model of the retailer and the manufacturer, respectively. Then, we develop the joint profit model of the manufacturer and the retailer. Subsequently, we apply the proposed models to derive a number of propositions regarding the returns policy in order to realize how the optimal decision will change when some parameters vary. Some proofs of the propositions are detailed in the Appendix.

The retailer's expected profit function is expressed as follows.

$E(\pi_r)=$ sales revenue – cost of goods sold – goodwill loss+ returns profit

$$=pE\{\min(Q,D)\}-wQ-\beta E\{\max(0,D-Q)\}+sE\{\max(0,Q-D)\} \quad (1)$$

The manufacturer's expected profit function is expressed as follows.

$E(\pi_m)=$ sales profit from the retailer – returns loss

$$=(w-c)Q-sE\{\max(0,Q-D)\} \quad (2)$$

Given a wholesale price and unit buyback price by the manufacturer, the retailer maximizes his own profit in order to determine the optimal order quantity.

$$\frac{\partial E(\pi_r)}{\partial Q}=(p+\beta-w)-(p+\beta-s)F(Q) \quad (3)$$

The second-order condition is strictly smaller than zero.

$$\frac{\partial^2 E(\pi_r)}{\partial Q^2} = -(p+\beta-s)\, f(Q) < 0 \quad (4)$$

In order to obtain the optimal order quantity, we let $\frac{\partial E(\pi_r)}{\partial Q} = 0$.

Hence, $F(Q) = \frac{p+\beta-w}{p+\beta-s}$ (5)

According to Eq. (5), the optimal order quantity, $Q^*$, is expressed as follows.

$$\begin{aligned} Q^* &= F^{-1}(\frac{p+\beta-w}{p+\beta-s}) \\ &= \mu - \sqrt{3}\,\sigma + 2\sqrt{3}\,\sigma(\frac{p+\beta-w}{p+\beta-s}) \\ &= \mu + \frac{(p+\beta+s-2w)\sqrt{3}\,\sigma}{p+\beta-s} \end{aligned} \quad (6)$$

Substituting Eq. (6) into Eq. (2), the manufacturer's expected profit becomes below.

$$E(\pi_m) = (w-c)[\mu + \frac{(p+\beta+s-2w)\sqrt{3}\sigma}{p+\beta-s}] - \frac{\sqrt{3}\sigma(p+\beta-w)^2 s}{(p+\beta-s)^2} \quad (7)$$

Given an order quantity by the retailer, the manufacturer maximizes his own profit in order to determine the optimal wholesale price.

$$\frac{\partial E(\pi_m)}{\partial w} = u + \frac{(p+\beta+s+2c-4w)\sqrt{3}\sigma}{p+\beta-s} + \frac{2\sqrt{3}\sigma(p+\beta-w)s}{(p+\beta-s)^2} \quad (8)$$

The second-order condition is strictly smaller than zero.

$$\frac{\partial^2 E(\pi_m)}{\partial w^2} = -\frac{4\sqrt{3}\sigma}{p+\beta-s} - \frac{2\sqrt{3}\sigma s}{(p+\beta-s)^2} < 0 \quad (9)$$

In order to obtain the optimal wholesale price, we set $\frac{\partial E(\pi_m)}{\partial w} = 0$.

Thus,

$$w^* = \frac{\mu(p+\beta-s)^2 + \sqrt{3}\sigma[(p+\beta)(p+\beta+2s+2c) - s(s+2c)]}{2\sqrt{3}\sigma(2p+2\beta-s)} \quad (10)$$

Combining Eq. (1) and Eq. (2), the retailer and manufacturer joint profit function becomes below.

$$E(\pi_j) = pE\{\min(Q,D)\} - cQ - \beta E\{\max(0, D-Q)\} \quad (11)$$

The retailer and manufacturer maximize their joint profit in order to determine the optimal order quantity.

$$\frac{\partial E(\pi_j)}{\partial Q} = (p+\beta-c) - (p+\beta)F(Q) \quad (12)$$

The second-order condition is strictly smaller than zero.

$$\frac{\partial^2 E(\pi_j)}{\partial Q^2} = -(p+\beta)\, f(Q) < 0 \quad (13)$$

In order to obtain the optimal order quantity, we set $\frac{\partial E(\pi_j)}{\partial Q} = 0$.

Hence, $F(Q) = \frac{p+\beta-c}{p+\beta}$ (14)

According to Eq. (14), the optimal order quantity, $Q_j^*$, is expressed as follows.

$$\begin{aligned} Q_j^* &= F^{-1}(\frac{p+\beta-c}{p+\beta}) \\ &= \mu - \sqrt{3}\,\sigma + 2\sqrt{3}\,\sigma(\frac{p+\beta-c}{p+\beta}) \\ &= \mu + \frac{(p+\beta-2c)\sqrt{3}\,\sigma}{p+\beta} \end{aligned} \quad (15)$$

Substituting Eq. (15) into Eq. (11), Eq. (11) becomes below.

$$E(\pi_j) = (p-c)\mu - \frac{c(p+\beta-c)\sqrt{3}\sigma}{p+\beta} \quad (16)$$

**Proposition 1** The retailer's optimal order quantity in the joint model will be larger than that in the individual model.

Proof:

Accounting to Eq. (15) and Eq. (6), $Q_j^* - Q^*$ equals Eq. (17).

$$\begin{aligned} Q_j^* - Q^* &= \frac{(p+\beta-2c)\sqrt{3}\sigma}{p+\beta} - \frac{(p+\beta+s-2w)\sqrt{3}\sigma}{p+\beta-s} \\ &= (\sqrt{3}\sigma - \frac{2c\sqrt{3}\sigma}{p+\beta}) - (\sqrt{3}\sigma + \frac{2(s-w)\sqrt{3}\sigma}{p+\beta-s}) \\ &= \frac{-2c\sqrt{3}\sigma(p+\beta-s) + 2(w-s)(p+\beta)\sqrt{3}\sigma}{(p+\beta)(p+\beta-s)} \\ &= \frac{2\sqrt{3}\sigma(p+\beta)(w-s-c) + 2sc\sqrt{3}\sigma}{(p+\beta)(p+\beta-s)} \end{aligned} \quad (17)$$

Since $(p+\beta)(p+\beta-s) > 0$, $(p+\beta) > 0$, $(w-s-c) > 0$, $s > 0$ and $c > 0$, $Q_j^* - Q^*$ is always positive. □

**Proposition 2** The retailer's optimal order quantity in the joint model will be twice as large as that in the individual model when $s = 0$.

Proof:

If $s = 0$, Eq. (15), Eq. (6) and Eq. (10) can be rewritten as Eqs. (18) - (20), respectively.

$$Q_{j,s=0}^* = \mu + \frac{(p+\beta-2c)\sqrt{3}\sigma}{p+\beta} \quad (18)$$

$$Q_{s=0}^* = \mu + \frac{(p+\beta-2w)\sqrt{3}\sigma}{p+\beta} \quad (19)$$

$$w_{s=0}^* = \frac{\mu(p+\beta) + \sqrt{3}\sigma(p+\beta+2c)}{4\sqrt{3}\sigma} \quad (20)$$

Thus,

$$\frac{Q_{j,s=0}^*}{Q_{s=0}^*} = \frac{\mu + \frac{(p+\beta-2c)\sqrt{3}\sigma}{p+\beta}}{\mu + \frac{(p+\beta-2w)\sqrt{3}\sigma}{p+\beta}} = \frac{(p+\beta)\mu + (p+\beta-2c)\sqrt{3}\sigma}{(p+\beta)\mu + (p+\beta-2w)\sqrt{3}\sigma} \quad (21)$$

Substituting Eq. (20) into the denominator term of Eq. (21) gives

$$\begin{aligned} &(p+\beta)\mu + (p+\beta-2w)\sqrt{3}\sigma \\ &= (p+\beta)\mu + (p+\beta)\sqrt{3}\sigma - \frac{\mu(p+\beta) + \sqrt{3}\sigma(p+\beta+2c)}{2}. \\ &= \frac{1}{2}[(p+\beta)\mu + (p+\beta-2c)\sqrt{3}\sigma] \end{aligned}$$

Thus,

$$\frac{Q^*_{j,s=0}}{Q^*_{s=0}} = \frac{(p+\beta)\mu+(p+\beta-2c)\sqrt{3}\sigma}{(p+\beta)\mu+(p+\beta-2w)\sqrt{3}\sigma}$$
$$= \frac{(p+\beta)\mu+(p+\beta-2c)\sqrt{3}\sigma}{0.5[(p+\beta)\mu+(p+\beta-2c)\sqrt{3}\sigma]} = 2 \quad \square$$

**Proposition 3** In the individual model, when the manufacturer offers the returns policy, the retailer's optimal order quantity will be more than that in a no-returns policy.
Proof:
According to Eq. (6) and Eq. (19) gives

$$Q^* - Q^*_{s=0} = (\mu + \frac{(p+\beta+s-2w)\sqrt{3}\,\sigma}{p+\beta-s}) - (\mu + \frac{(p+\beta-2w)\sqrt{3}\sigma}{p+\beta})$$
$$= \frac{\sqrt{3}\sigma[(p+\beta)(p+\beta+s-w)-(p+\beta-2w)(p+\beta-s)]}{(p+\beta-s)(p+\beta)}$$
$$= \frac{\sqrt{3}\sigma[2s(p+\beta-w)]}{(p+\beta-s)(p+\beta)}$$

Since $p > w > s > 0$ and $\beta > 0$

Thus $Q^* - Q^*_{s=0} = \frac{\sqrt{3}\sigma[2s(p+\beta-w)]}{(p+\beta-s)(p+\beta)} > 0 \quad \square$

**Proposition 4** In the joint model, the retailer's optimal order quantity in the return scenario will be the same as that in the no-returns scenario.
Proof:
If $s=0$, Eq. (1) and Eq. (2) will reduce to Eq. (22) and Eq. (23), respectively.

$$E(\pi_{r,s=0}) = pE\{\min(Q,D)\} - wQ - \beta E\{\max(0, D-Q)\} \quad (22)$$

$$E(\pi_{m,s=0}) = (w-c)Q \quad (23)$$

Combining Eq. (22) and Eq. (23), the retailer and manufacturer joint profit function is equivalent to Eq. (24).

$$E(\pi_{r,s=0}) + E(\pi_{m,s=0}) = pE\{\min(Q,D)\} - cQ - \beta E\{\max(0, D-Q)\} \quad (24)$$

Since Eq. (24) is equals to Eq. (11), the optimal order quantity is the same before and after returns. □

**Proposition 5** $Q^*_j = Q^*_{j,s=0} > Q^* > Q^*_{s=0} > 0$

According to the above mentioned propositions, proposition 1 shows that $Q^*_j - Q^* > 0$. Then proposition 3 shows that $Q^* - Q^*_{s=0} > 0$ and proposition 4 shows that $Q^*_j = Q^*_{j,s=0}$. Thus, $Q^*_j = Q^*_{j,s=0} > Q^* > Q^*_{s=0} > 0$.□

## 4. DISCUSSION

Comparing the retailer's optimal order quantity with individual supply chain and whole supply chain, show that the latter is always larger. Next, we note a special phenomenon that if the returns policy is neglected, the order quantity with whole supply chain will become twice as large as that with individual supply chain. Additionally, we can observe the result which illustrates that the optimal order quantity in the whole supply chain is indeed more than that in the individual supply chain.

Comparing the returns policy with the no-returns policy, the retailer does not always orders more goods when the manufacturer provides a returns policy. The retailer orders higher quantities only in the two parties' self-interested manner. Our results show that the optimal order quantity with returns policy is more than that without returns policy under the non-coordinating model. On the other hand, if the manufacturer and the retailer can coordinate with each other and share information with respect to the cost structure, then the optimal order quantity of the retailer is the same before and after return. This implies that if both the returns policy and the coordination mechanism are adopted simultaneously, the returns policy becomes irrelevant. According to these observations, we can conclude that the returns policy is relevant and positive factor in the individual model, whereas it becomes irrelevant in the coordinated model.

Finally, the results for all different scenarios are summarized as follows. The coordinated policy is superior to the non-coordinated policy, regardless of whether the returns policy is provided or not. In addition, comparing the returns policy with the no-returns policy, the former is better than the latter when the coordination mechanism is not adopted. On the contrary, there are no differences between the two policies when the coordination mechanism exists. Consequently, the manufacturer can make the appropriate decision depending on the different policies.

## 5. CONCLUSION

In this study, we investigate the impact of the returns policy. We derive an analytical model and present some proofs to test the propositions regarding returns policy. First, from the analytical results, we compare the impact of order decision with individual channel and coordinating channel, and also discuss the difference of the ordering decision between returns policy and no-returns policy.

Our analytical results provide more complete examinations and present a number of meaningful managerial insights on how a decision maker should implement the firm's returns policy. We suggest that the manufacturer and retailer should share information of the cost structure with each other. If they are willing to adopt a coordinated strategy, the returns policy will become irrelevant. However, if only both sides consider their own expected profits, sum of the two parties' expected profits with returns policy will be higher than that with no-returns policy.

In conclusion, we suggest that both the manufacturer and the retailer will be benefited when the manufacturer offers the returns policy or when the two parties coordinate with each other. However, once both the returns policy and the coordination mechanism are adopted simultaneously, the returns policy becomes irrelevant.

## Appendix

Some uniform distribution formulas

If $D \sim U(a,b)$, $\mu = \frac{a+b}{2}$, $\sigma = \frac{(b-a)}{2\sqrt{3}}$

Thus,

$a = \mu - \sqrt{3}\sigma$, $b = \mu + \sqrt{3}\sigma$

probability density function (pdf): $f(x) = \frac{1}{b-a}$

cumulative distribution function (cdf): $F(x) = \frac{x-a}{b-a} = \frac{x-\mu+\sqrt{3}\sigma}{2\sqrt{3}\sigma}$

inverse cdf: $F^{-1}(x) = \mu - \sqrt{3}\sigma + 2\sqrt{3}\sigma x$

$$\begin{aligned} E(A) &= E\{\min(D,Q)\} \\ &= \int_0^Q x\, f(x)\,dx + \int_Q^{+\infty} Q\, f(x)\,dx \\ &= Q - \frac{(Q-a)^2}{2(b-a)} \\ &= Q - \frac{(Q-\mu+\sqrt{3}\sigma)^2}{4\sqrt{3}\sigma} \end{aligned}$$

$$\begin{aligned} E\{\max(0,D-Q)\} &= E(D) - E(A) \\ &= \mu - Q + \frac{(Q-\mu+\sqrt{3}\sigma)^2}{4\sqrt{3}\sigma} \end{aligned}$$

$$\begin{aligned} E\{\max(0,Q-D)\} &= Q - E(A) \\ &= \frac{(Q-u+\sqrt{3}\sigma)^2}{4\sqrt{3}\sigma} \end{aligned}$$

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