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Study and Analysis of Fat-Tail Risk & Other Statistical Properties of Indian Stock Market

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Abstract- *In this paper, the existence of fat-tail risk and other statistical properties of the Indian Stock Market are studied and analyzed. A fat-tail risk in financial markets refers to extreme swings in the markets. The daily data of S&P CNX Nifty during the years 2001 to 2009 is selected for experiment. The actual distribution of returns are examined which exhibit the statistical characteristics of leptokurtosis and the frequencies of distributions falling into certain ranges of daily returns are compared. The daily return of stock index is forecasted by Fuzzy Time Series Model. The statistical study has shown that the Indian Stock Market has more volatile or risky trading environment than normal distribution.*

Keyword: Fat-tail risk; Fuzzy Time Series Model; Leptokurtosis; Normal distribution; volatility.

1. INTRODUCTION

The stock prices play an important role to study the market fluctuations. The investor and other stakeholders of financial market want to uncover hidden patterns and predict future trends. This has fascinated and attracted many mathematicians to unveil the mystery of financial market. The different types of mathematical and statistical models with varying degree have been discussed. Every stock market investor wants to maximize their returns by buying or selling their shares at an appropriate time. The greater returns on investment associated with greater market risk. The stock market is a very efficient market place and there is always risk to investing money in the market. Beta and standard deviation are two tools commonly used to measure stock risk.

Recently, some research work has been done to investigate the statistical properties of fluctuation of stock prices in a stock market [1, 3, 5, 7, 9, 10, 11, 12, 14, 15].Their research work showed that the stock price change behave according to a random Gaussians or normal distribution for long time intervals but to deviate from it for short time steps, especially the deviation appears at the tail part of the distribution usually called the fat-tail phenomena. A fat tail is a property of probability distributions exhibiting extremely large kurtosis. In mathematical term, the condition of probability distribution that exhibits fat tail is called leptokurtosis.

In the present paper our study focuses on the fat tail risk of return and other statistical properties of stock market. The data base which is used in the present paper is from the website of NSE (www.nseindia.com). The data of the daily closing prices of eight year period during the year 2001 to 2009 is selected for study. The total number of observed data for the stock prices is about 2245. We also forecast the predicted rate of return by the fuzzy time series model.

2. METHODOLOGY

To study the fat-tail risk in the Indian stock market we calculate the daily returns of the S&P CNX Nifty and compute statistics to examine the daily rate of return of market.

Mathematically Rate of Return is calculated as:

$$\text{Rate of Return} = \frac{p_t - p_{t-1}}{p_{t-1}} * 100 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where P_t = Stock price in time period t

P_{t-1} = Stock price in time period t-1

Skewness and kurtosis are the important statistics used to describe stock market data. [4] Skewness describes the asymmetry of the probability of a real valued random variable. When the value of skewness equals zero, the distribution is symmetric, when the value of skewness is greater than zero, the distribution is positive skewness and when the value of skewness is less than zero, the distribution is negative skewness. Kurtosis describes the peakedness of the probability distribution of a real valued random variable. A high kurtosis distribution has a sharper peak and longer, fatter tails while a low kurtosis distribution has a more rounded peak and shorter thinner tails. Distributions with zero excess kurtosis are called mesokurtic. A distribution with positive excess kurtosis is called leptokurtic and a distribution with negative excess return is called platykurtic.

Mathematically skewness and kurtosis is defined as:

$$\text{Skewness} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^3 / S^3 \quad \ldots\ldots\ldots\ldots (2)$$

$$\text{Kurtosis} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^3 / S^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $\bar{x}$ is the mean and S is the standard deviation.

Beta coefficient is also important parameter to describe the stock market. It is also referred as financial elasticity or correlated relative volatility and can be referred to as a measure of the sensitivity of the asset's return to market returns. On an individual asset level, measuring beta can give clues to volatility and liquidity in the market place. An asset with a beta of zero means that its price is not at all correlated with the market. A positive beta means that the asset generally follows the market. A negative beta shows that the asset inversely follows the market.

The formula for the beta of an asset is:

Beta = Covariance (stock vs. market returns) / Variance (market)…… (4)

ANOVA [4] is a general technique that can be used in our experiment to test the hypothesis that the mean among two or more stocks are equal under the assumption that the sampled populations are normally distributed. We have randomly selected three stocks and found the rate of return of each stock. We have tested whether any of the stocks have a different rate of return from the other, at 0.05% level of significance. We have taken the following hypothesis:

Ho: All the stocks have same average rate of return

H1: All the stocks don't have the same rate of return

For Forecasting the rate of return fuzzy time series model is used. FTS is one of the computational intelligence methods and it is successfully applied in the area of stock market. The details of FTS are given in [2, 8, and 13]

3. RESULT AND DISCUSSION

The statistics of the S&P Nifty data, including Mean (0.0786%), Standard Deviation (1.7429%), Kurtosis (8.63) and Skewness (-0.03). The Kurtosis- a measure of Peakedness- of a normal distribution is zero with a kurtosis of 8.63 indicate that the distribution has fat tail. The skewness is a measure of the asymmetry in the distribution curve with negative skewness indicating that the curve has a longer left (or negative) tail. That is, there are a number of extreme negative daily returns than extreme positive daily returns.

Table1: Descriptive Statistics for S&P CNX Nifty Daily Return Data (2001~ 2009)

Sample Number	2245
Mean	0.0786%
Standard Deviation	1.7429%
Kurtosis	8.63
Skewness	-0.03
Maximum Daily Return	-12.2377
Minimum Daily Return	17.7441

Based on the statistics in Table 1, we classified the S&P CNX Nifty Daily Return into 13 categories as shown in Table 2. The more +/-6 σ we observe, the more evidently Fat-tail Risk exists in the daily return.

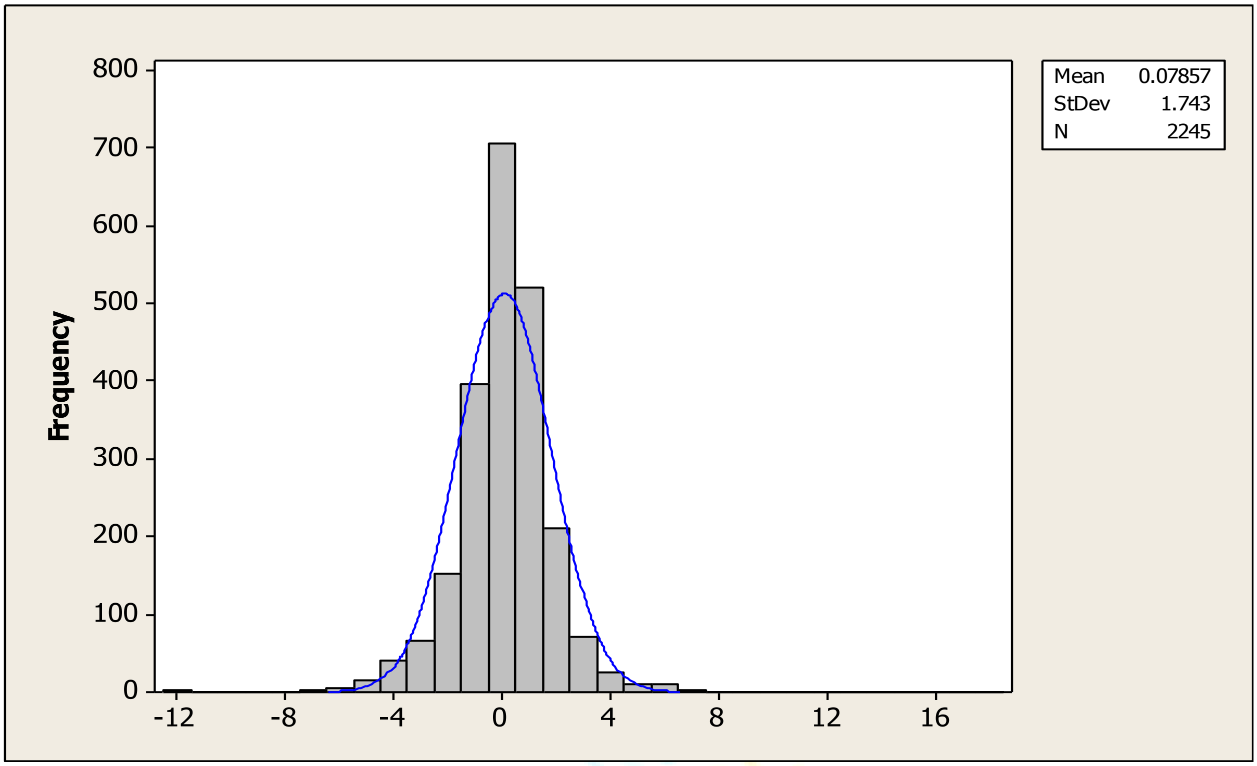
Table2: S&P CNX Nifty Daily Return – Standard Deviation Ranges

Standard Deviations from Mean	Daily Return Range
$+6\sigma$	Above +10.46 %
$+5\sigma$	+8.72 ~ +10.46%
$+4\sigma$	+7.0 ~ +8.72%
$+3\sigma$	+5.23 ~ +7.0%
$+2\sigma$	+3.49 ~ +5.23%
$+1\sigma$	+1.75 ~ +3.49%
0σ	-1.12 ~ +1.75%
-1σ	-1.75 ~ -1.12%
-2σ	-3.49 ~ -1.75%
-3σ	-5.25 ~ -3.49%
-4σ	-7.0 ~ -5.23%
-5σ	-8.72 ~ -7.0%
-6σ	Below -8.72%

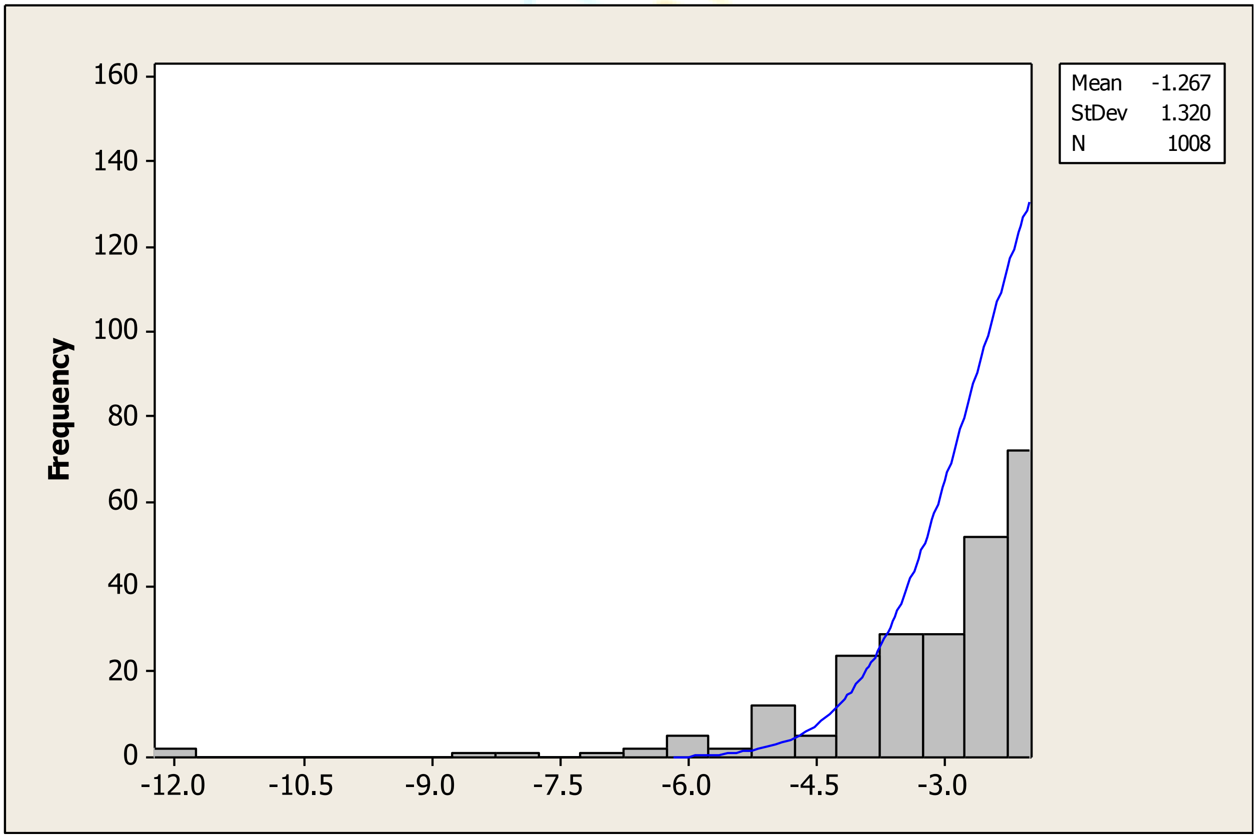
Under the statistical normal distribution of performance returns, deviations from the mean return should occur with a certain frequency; the larger the deviation the lower the frequency. Table 3 shows that for the daily performance of the S&P CNX Nifty data of actual distribution.

Table 3: S&P CNX Nifty data - Actual Distribution

	Actual Distribution	
Standard Deviations from Mean	Observed Frequency	Percentage
$+6\sigma$	1	0.044543%
$+5\sigma$	0	0%
$+4\sigma$	1	0.044543%
$+3\sigma$	14	0.623608%
$+2\sigma$	35	1.55902%
$+1\sigma$	210	9.35412%
0σ	1584	70.55679%
-1σ	20	0.890869%
-2σ	312	13.89755%
-3σ	53	2.360802%
-4σ	11	0.489978%
-5σ	2	0.089087%
-6σ	2	0.089087%
Total	**2245**	**100%**



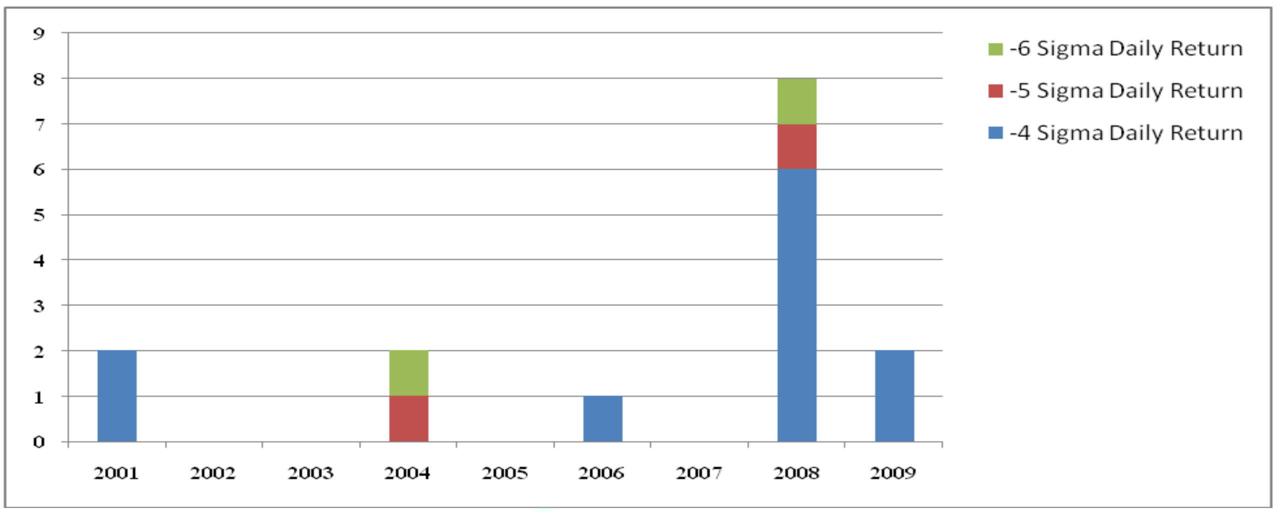
Graph 1: Actual S&P Daily Returns vs Normal Distribution



Graph2: Actual S&P Daily Returns vs Normal Distribution (from -3% to -12%)

Table 4 shows the historical trend of the Fat-tail risk.

Year	Return	-4 σ Daily Return		-5 σ Daily Return		-6 σ Daily Return	
2001	-13.6446%	2	18.2%	0%	0%	0	0%
2002	4.608815%	0	0%	0%	0%	0	0%
2003	56.15562%	0	0%	0%	0%	0	0%
2004	14.04108%	0	0%	1%	50%	1	50%
2005	32.56634%	0	0%	0%	0%	0	0%
2006	37.96679%	1	9.09%	0%	0%	0	0%
2007	46.89357%	0	0%	0%	0%	0	0%
2008	-63.2515%	6	54.55%	1%	50%	1	50%
2009	62.09265%	2	18.2%	0%	0%	0	0%
Total		**11**		**2**		**2**	



Graph 3: Graphical representation of fat-tail days in selected years.

Beta values are based on past performances may not be accurate to predict the future, and market volatility. Beta is important factor to consider before making a trading decision. Beta coefficient reflects the future relative movements of stock vs. market index. We have randomly selected three stocks (ABB, BPCL and INFOSYS). On the basis of beta coefficient we have found the risk/volatility of the market. Generally, the value of 1 is given to the market.

Table 5 shows the beta value of ABB, BPCL and INFOSYS

Stocks	Beta Coefficient
ABB	0.76003117
BPCL	0.659619929
INFOSYS	0.777863292

Stock ABB, BPCL and INFOSYS has a beta value 0.76, 0.66, 0.78. This means that the potential return of these stocks is equal to 76%, 66% and 78% of that of market as a whole.

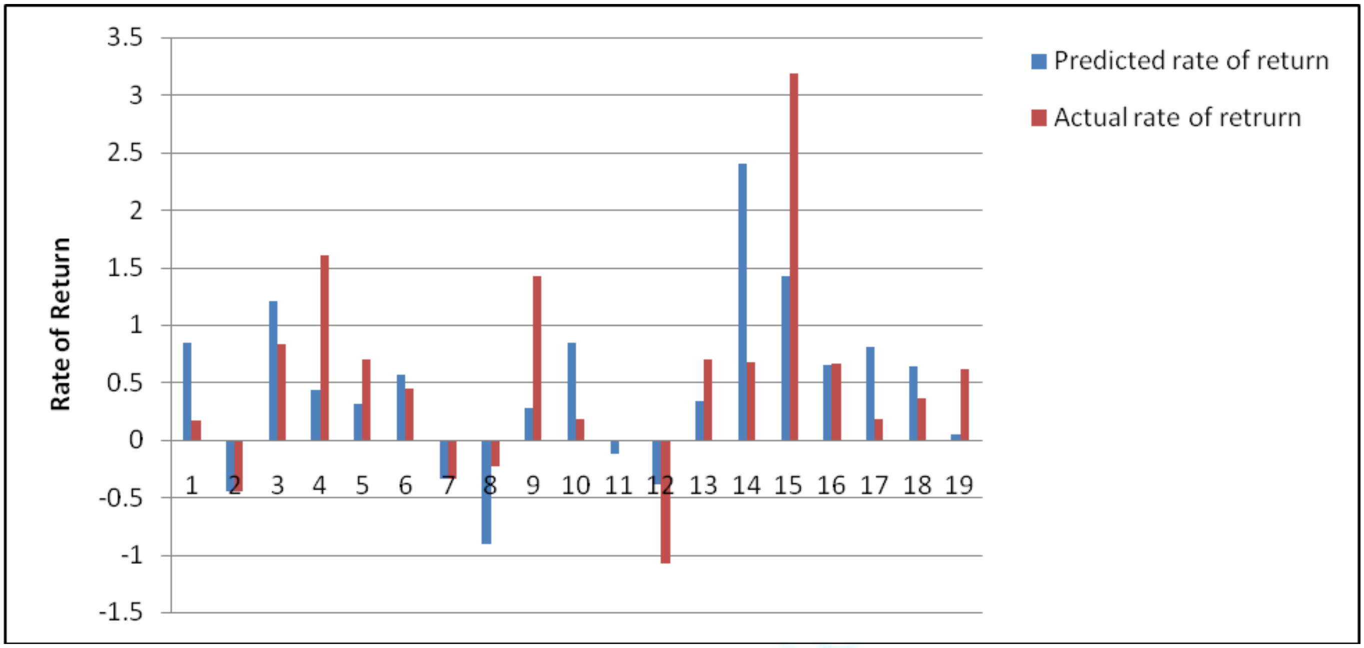
Next we used ANOVA to found whether any of the stocks have a different rate of return from the others. Table 6 shows the ANOVA table of three stocks

Table 6: ANOVA table

Source	DF	SS	MS	F	p
Between groups(or factor)	3	0.000242	0.000081	0.38	0.771
Within groups (or error)	7984	1.719380	0.000215		
Total	7987	1.719622			

The F statistic is 0.38. On the basis of ANOVA test we accept the null hypothesis at 5% level of significant and we concluded that all the stocks have same average rate of return.

Again we found the future rate of return of NSE. For that we used fuzzy time series model.



Graph 4: Actual versus Predicted rate of return of NSE

Average forecasting error of FTS is calculated by $\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right|$ and it is 1.96. It means that this method gives 98.04% accuracy.

4. CONCLUSION

The purpose of this paper was to examine Fat tail risk in Indian Stock Market. When compared to a normal distribution, historical data has shown significant degree of such risk in the returns of the S&P CNX Nifty Daily return data. This study has shown that Indian Stock market is more volatile than assumed by a simple normal distribution. Beta coefficient of three stocks shows that all the three assets follow the market index. ANOVA table also shows that all stocks have average rate of return. Fuzzy time series model helps to forecast the rate of return of Indian Stock market.

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Author's Biography

Dr. Jyoti Badge (1st April 1979) has done her Ph. D. in Mathematics from Maulana Azad National Institute of Technology, Bhopal in 2012. Currently she is working as an Assistant Professor in ***Institute of Professional Education and Research (IPER), Bhopal***. Her research interest includes financial mathematics, data mining and portfolio optimization. She is a life time member of Gwalior Academy of Mathematical Science (GAMS).